A predator-prey model with parasitic infection of the predator

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Introduction

A predator-prey model incorporating parasitic infection of the predator is constructed. In contrast to similar models in the literature, the distribution of the parasites within hosts is considered explicitly. Equilibria of the system corresponding to parasite-present and parasite-free states are discussed, and an example of successful parasitic invasion is presented. Areas of future study and analysis are discussed at the end.

Model formulation

The predator-prey model underlying the epidemiological system is a simple Lotka-Volterra model with logistic growth of the prey. The predator and prey populations are denoted by X and Y, respectively. A third compartment is added to account for the population dynamics of a within-host parasite (denoted by P). Consideration of an infective larvae/free-living stage of the parasite is omitted. As is the case with many macroparasites, the distribution per host is assumed to follow a negative binomial distribution with clumping parameter k and mean P/X. The prey is unaffected by the parasite and does not serve as an intermediate host. Rather, the parasite is ingested by the predator at a rate proportional to the predator population and the number of within-host parasites in the system. Parasite fecundity is density-independent, while parasite-induced host mortality is assumed to be density-dependent with pathogenicity γ. The system of ODEs representing this system is below, while a schematic is given in Figure 1.

$$\dot{X} = c\delta XY - \mu_1 X - \gamma P$$
$$\dot{Y} = \tau Y\left(1 - \frac{Y}{K}\right) - \delta XY - \mu_2 Y$$
$$\dot{P} = P\left(\lambda dX - (\mu_1 + \mu_3 + \gamma) - \frac{\gamma(k+1)}{k}\frac{P}{X}\right)$$

Results

Of interest to any epidemiological system is the basic reproduction number. For a macroparasite model, the reproduction number is the average number of offspring that reach sexual maturity per adult worm absent density-dependent limitations. Since

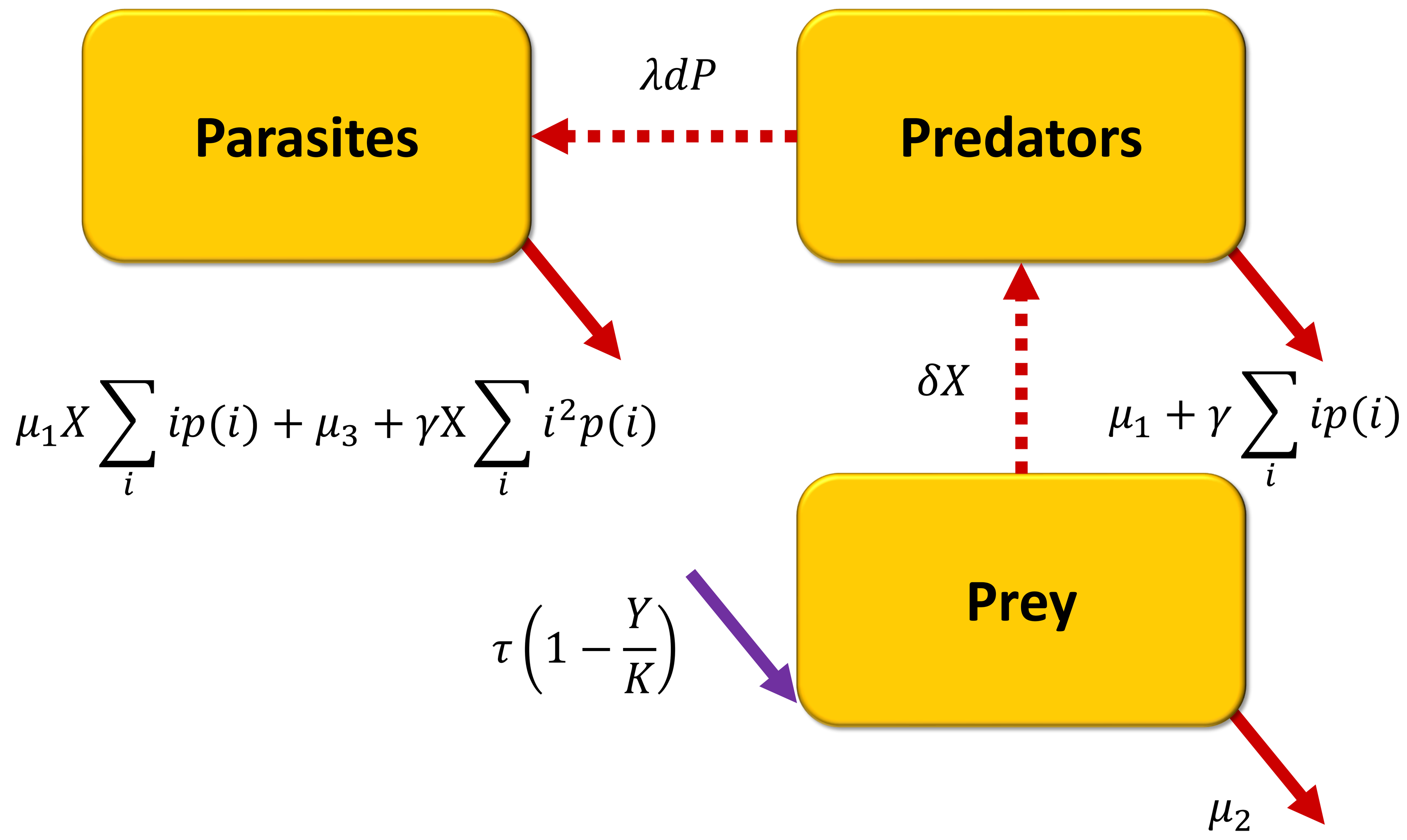

Figure 1. Schematic showing key dynamics of the ODE model. Dashed arrows indicate subsistence, red solid arrows denote death, and purple solid arrows denote birth. Note that the prey is the only population not reliant on another population. Rates are given next to their corresponding arrows. The distribution of parasites is explicitly given, where i denotes the number of parasites and $p(i)$ is the probability of a predator possessing i parasites.

density-dependence is a key consideration of the above model, however, this quantification is not particularly useful and unenlightening. Nonetheless, the model possesses two physically relevant equilibria, corresponding to states in which the parasite persists or goes extinct. For either fixed point, we require that $\mu_2 < \tau$ to ensure survival of the prey population in the absence of predation. The following inequalities must be satisfied in order to guarantee the existence of a positive coexistence equilibrium where all populations in the model persist:

$$kd\lambda(\mu_2 - \tau) < M < 0,$$
$$cK\delta^2(\mu_1 + \gamma + \mu_3) < d\lambda(cK\delta(\tau - \mu_2) - \tau\mu_1),$$

where $M = \delta(\mu_1 - k(\gamma + \mu_3))$. Stability analysis of the coexistence equilibrium was analytically unwieldy. Analysis of the parasite-free equilibrium was expectedly more tractable, especially since this equilibrium coincides exactly with the coexistence equilibrium of the Lotka-Volterra model with logistic growth of prey.

Discussion

Further analysis is needed to fully explore the dynamics of the system, especially in predicting when parasitic invasion of the predator population is successful. By numerical experiments alone, it is easily shown that there are certain conditions under which, when introduced to the population, the parasite can near-eradicate the predator population. This could be useful in scenarios where the prey population would have otherwise been driven to extinction by the predator. An example of successful parasitic invasion is shown in Figure 2, where the trajectories of a parasite-free and parasite-present system with initial condition $(X_0, Y_0, P_0) = (25, 50, 100)$ are compared.

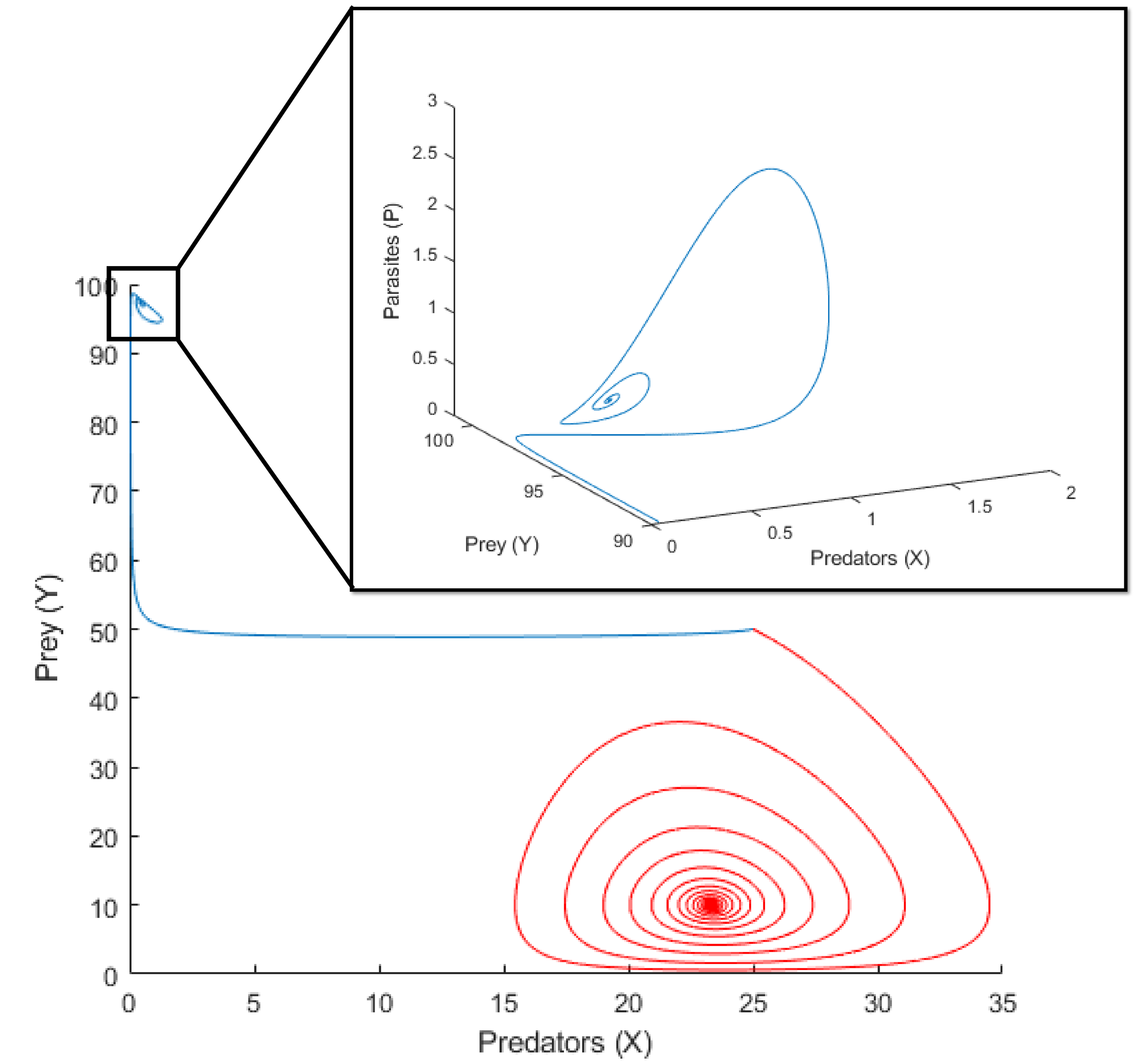

Figure 2. Plot of the predator and prey populations with and without parasitic infection, the former scenario is plotted in blue while the latter in red. Parameter values used: $\delta = 0.05$, $\mu_1 = 0.05$, $\gamma = 0.5$, $\tau = 1.2$, $K = 100$, $\mu_2 = 0.05$, $\lambda = 6$, $d = 0.5$, $\mu_3 = 0.1$, $k = 1.44$, $c = 0.1$.